

3.12 Multiple Angle Identities

Name _____ Date _____ Period _____

Find the exact value of each expression using double-angle identities.

1. $\sin(90^\circ)$

2. $\tan(60^\circ)$

3. $\cos\left(\frac{4\pi}{3}\right)$

4. $\sin\left(\frac{2\pi}{3}\right)$

Find the exact value of each expression using half-angle identities.

5. $\sin(15^\circ)$

6. $\tan(15^\circ)$

7. $\cos\left(\frac{\pi}{8}\right)$

8. $\sin(22.5^\circ)$

For each equation determine whether the positive or negative sign makes the equation correct. Do not use a calculator.

9. $\sin(118.5^\circ) = \pm \sqrt{\frac{1 - \cos(237^\circ)}{2}}$

10. $\cos(100^\circ) = \pm \sqrt{\frac{1 + \cos(200^\circ)}{2}}$

11. $\cos\left(\frac{9\pi}{7}\right) = \pm \sqrt{\frac{1 + \cos\left(\frac{18\pi}{7}\right)}{2}}$

12. $\tan\left(\frac{17\pi}{12}\right) = \pm \sqrt{\frac{1 - \cos\left(\frac{17\pi}{6}\right)}{1 + \cos\left(\frac{17\pi}{6}\right)}}$

Use identities to simplify each expression. Do not use a calculator.

13. $2\sin(13^\circ)\cos(13^\circ)$

14. $2\cos^2(22.5^\circ) - 1$

15. $\frac{\tan(15^\circ)}{1 - \tan^2(15^\circ)}$

16. $\cos^2\left(\frac{\pi}{9}\right) - \sin^2\left(\frac{\pi}{9}\right)$

Prove that each equation is an identity.

17. $\cos^4 x - \sin^4 x = \cos 2x$

18. $(\sin x - \cos x)^2 = 1 - \sin 2x$

19. $\frac{\cos 2x}{\sin^2 x} = \csc^2 x - 2$

20. $2\sin^2\left(\frac{x}{2}\right) = \frac{\sin^2 x}{1 + \cos x}$

In each case, find $\sin \alpha$, $\cos \alpha$, $\tan \alpha$, $\csc \alpha$, $\sec \alpha$, and $\cot \alpha$.

21. $\cos 2\alpha = \frac{3}{5}$ and $0^\circ < 2\alpha < 90^\circ$

22. $\sin\left(\frac{\alpha}{2}\right) = \frac{4}{5}$ and $\frac{\alpha}{2}$ is in quadrant II

Solve each problem.

23. Find the exact value of $\sin(2\alpha)$ given that $\sin(\alpha) = \frac{3}{5}$ and α is in quadrant II.

24. Find the exact value of $\cos(2\alpha)$ given that $\sin(\alpha) = \frac{8}{17}$ and α is in quadrant II.

25. Find the exact value of $\cos \beta$ if $\sin \beta = \frac{2}{3}$ and β is in quadrant II.